Body-wise analysis of reaction forces in overconstrained multibody systems

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EXTENDED ABSTRACT

1 Introduction

The knowledge of load distribution within a multibody system is crucial for designers or analysts. Multibody simulation methods provide tools for acquiring necessary information. However, when relatively simple rigid-body models are used to analyze overconstrained mechanisms, the determination of reaction forces is hampered, as some of the calculated reaction forces are non-unique [1]. Usually, in the case of non-unique reaction components, the multibody software provides a solution that results from the adopted—purely mathematical and not embedded in physics—method of handling redundant constraints [2]. It is crucial for software users to know that some of the acquired reaction solutions are unrealistic. To this end, appropriate methods of reaction solvability analysis have been developed [2, 3]. These methods examine the selected subset of constraints and reveal whether or not the generalized reactions that correspond to the analyzed subset can be uniquely determined. So far, however, the uniqueness is analyzed only “constraint-wise” since the generalized constraint reaction force (unique or not) is related to the whole multibody system rather than its individual bodies. Hence, it could be useful to propose a “body-wise” perspective for the uniqueness analysis.

To formulate the body-wise approach, let us consider a generic overconstrained multibody system schematically depicted in Figure 1. Note that for the purpose of dynamic analysis of various mechanisms (e.g., parallel robots, vehicles, etc.), some of the bodies can be considered as the main ones (e.g., the end-effector, the hull, etc.) and be of particular interest during the analysis. In Figure 1, these bodies are numbered 1-4. Auxiliary linkages that connect the main bodies are also presented, but the numbering of their parts is omitted. Let us assume that the subset of constraints representing the kinematic pairs indicated by the red color in Figure 1 was analyzed, and the non-uniqueness of the corresponding generalized reaction was detected. Mind that this generalized reaction jointly represents reactions acting on body 1 as well as on bodies (marked by the blue color) belonging to the auxiliary linkages. It would be useful to know whether, for the analyzed subset of constraints, the resultant reactions acting on body 1 are unique (note that non-unique components may sum up to a unique resultant). In other words, it would be useful to know whether the loads transmitted via the auxiliary linkages from body 2 to body 1 are unique. The proposed body-wise analysis is focused on checking whether the resultant reaction, acting on the specified body, and originating from the analyzed subset of constraints, is unique.

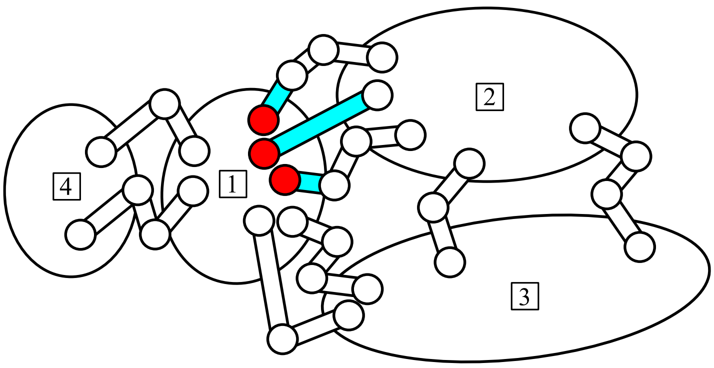

Figure 1: A generic overconstrained multibody system

2 Proposed method

The developed body-wise approach is an enhancement to the existing methods for analyzing the constraint reactions’ solvability. These methods consist in investigating the properties of the matrix $\mathbf{J}^T$, which is the transposed constraint Jacobian matrix (in the case of geometric/holonomic constraints) or—more generally—the transposed constraint matrix (if linear nonholonomic constraints are also present). It is assumed that absolute (Cartesian) coordinates are used to describe the multibody system.

At the beginning of the uniqueness test, the analyzed subset of constraints is designated according to our needs. Then, the transposed constraint matrix is rearranged and divided into parts corresponding to the analyzed subset (*A*) and to the remaining constraints (*R*). Such a procedure may be done as follows:

$$\mathbf{J}^T\mathbf{D} = \mathbf{J}^T\begin{bmatrix}\mathbf{D}_A & \mathbf{D}_R\end{bmatrix} = \begin{bmatrix}\mathbf{J}_A^T & \mathbf{J}_R^T\end{bmatrix}, \tag{1}$$

where **D** is an orthogonal permutation matrix, while $\mathbf{D}_A$ and $\mathbf{D}_R$ are its submatrices corresponding to the analyzed subset *A* and the remaining elements, respectively.

One of several methods can be used to conduct the analysis [2]. For example, when the *QR*-based method is employed, the constraint matrix is subjected to the *QR* decomposition:

$$\mathbf{J}\,\mathbf{E} = \mathbf{Q}\,\mathbf{R}\,, \tag{2}$$

where **E** is an orthogonal permutation matrix, **Q** is an orthogonal matrix, and **R** is an upper-trapezoidal matrix. The permutation matrix is used to ensure the proper arrangement of elements of matrix **R**—the last (*m–r*) rows of this matrix are zero (*m* is the number of constraints imposed on the multibody system, whereas *r* is the number of redundant constraints).

Then, to check the uniqueness of reactions corresponding to the analyzed subset of constraints, an auxiliary matrix $\mathbf{B}_A$ is evaluated:

$$\mathbf{B}_A = \mathbf{J}^T\mathbf{D}_A\mathbf{D}_A^T\mathbf{Q}\,. \tag{3}$$

It was proved that the generalized reactions associated with constraints from the analyzed subset *A* are unique if the last (*m–r*) columns of matrix $\mathbf{B}_A$ (further denoted as $\mathbf{B}_A^N$) consist of zeros only.

To introduce the body-wise enhancement to the outlined procedures, an additional matrix $\mathbf{H}^i$ must be defined. This matrix relates the generalized reactions with the reaction forces, torques, or both acting on the individual body *i*. In the case of investigating the resultant forces and torques simultaneously, the matrix takes the form:

$$\mathbf{H}^i = \begin{bmatrix}\mathbf{0}_{6\times 6(i-1)} & \mathbf{I}_{6\times 6} & \mathbf{0}_{6\times(n-6i)}\end{bmatrix}, \tag{4}$$

where **0** is the zero matrix, **I** is the identity matrix, and *n* is the number of coordinates.

It can be shown that the analyzed subset of constraints exerts on the body *i* unique resultant reaction forces and torques when the following condition is fulfilled:

$$\mathbf{H}^i\,\mathbf{B}_A^N = \mathbf{0}\,. \tag{5}$$

Similar enhancements, leading to analogous conditions, can be introduced to other than *QR*-based methods.

3 Conclusions

The proposed method of body-wise joint reactions solvability analysis is an extension of the methods existing so far. It provides designers or analysts with qualitatively new information regarding the properties of overconstrained mechanisms being investigated.

Acknowledgments

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References

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